Grade 6 Math Unit 1 - Ratios and Proportional Relationships

UNIT OVERVIEW

In Grade 6, instructional time should focus on four critical areas. This unit is connected to **Critical Focus Area #1**, Connecting ratio and rate to whole number multiplication and division using concepts of ratio and rate to solve problems. (See Connections for explanation)

This unit will address the following cluster:

- Understand ratio concepts and use ratio reasoning to solve problems

Students will understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. Use ratios, proportions, percents, and unit rates to solve real-world math problems including units of measure. Utilize tables of equivalent ratios, tape diagrams, double number line diagrams, coordinate planes, and equations to develop reasoning skills.

STANDARDS

CC_Common Core State Standards - Mathematics (2010) - Grade 6

Domain 6.RP Ratios and Proportional Relationships

Cluster Statement Understand ratio concepts and use ratio reasoning to solve problems.

Standard 6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.

Standard 6.RP.2 Understand the concept of a unit rate a/b associated with a ratio a:b with b is not equal to 0, and use rate language in the context of a ratio relationship.

Standard 6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

6.RP.3.a Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

6.RP.3.b Solve unit rate problems including those involving unit pricing and constant speed.

6.RP.3.c Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

6.RP.3.d Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

Domain 6.NS The Number System

Cluster Statement Compute fluently with multi-digit numbers and find common factors and multiples.

Standard 6.NS.4 Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor.

CONTENT ELABORATIONS

6.RP.1	A ratio is a comparison of two quantities or measures. The comparison can be part-to-whole (ratio of guppies to all fish in an aquarium) or part-to-part (ratio of guppies to goldfish). Students need to understand each of these ratios when expressed in the following forms: *6 to 15 or 6:15* . The values can be simplified to *2 to 5 or 2:5* ; however, *students would need to understand how the simplified values relate to the original numbers* . A rate is a ratio where two measurements are related to each other. When discussing measurement of different units, the word rate is used rather than ratio. Understanding rate, however, is complicated and there is no universally accepted definition. When using the term rate, contextual understanding is critical. Students need many opportunities to use models to demonstrate the relationship between quantities before they are expected to work with rate numerically. *Example:* A comparison of 8 black circles to 4 white circles can be written as the ratio of 8:4 and can be regrouped into 4 black circles to 2 white circles (4:2) and 2 black circles to 1 white circle (2:1). Students should be able to identify all these ratios and describe them using, "For every..., there are..." **MP.2, MP.6** should be emphasized.
6.RP.2	A unit rate expresses a ratio as part-to-one or one unit of another quantity. For example, if there are 2 cookies for 3 students, each student receives 2/3 of a cookie, so the unit rate is 2/3:1. If a car travels 240 miles in 4 hours, the car travels 60 miles per hour (60:1). Students understand the uni rate from various contextual situations. Students will often use unit rates to solving missing value problems. Cost per item or distance per time unit are common unit rates, however, students should be able to flexibly use unit rates to name the amount of either quantity in terms of the other quantity. Students will begin to notice that related unit rates are reciprocals as in the first example. It is not intended that this be taught as an algorithm or rule because at this level, students should primarily use reasoning to find these unit rates. In Grade 6, students are not expected to work with unit rates expressed as complex fractions. Both the numerator and denominator of the original ratio will be whole numbers. *Example:* On a bicycle you can travel 20 miles in 4 hours. What are the unit rates in this situation, (the distance you can travel in 1 hour and the amount of time required to travel 1 mile)? *Solution:* You can travel 5 miles in 1 hour written 5mi./1hr. and it takes 1/5 of an hour to travel each mile written 1/5hr./1mi. **MP.2, MP.6** should be emphasized.
6.RP.3a	**6.RP.3a** Ratios and rates can be used in ratio tables and graphs to solve problems. Previously, students have used additive reasoning in tables to solve problems. To begin the shift to proportional reasoning, students need to begin using multiplicative reasoning. To aid in the development of proportional reasoning the cross-product algorithm is not expected at this level. When working with ratio tables and graphs, whole number measurements are the expectation for this standard. To help understand the multiplicative relationship, students write equations. Writing equations is foundational for work in 7th grade. Numbers in a table can be expressed as ordered pairs and plotted on a coordinate plane.
6.RP.3b	Students recognize the use of ratios, unit rate and multiplication in solving problems, which could allow for the use of fractions and decimals. Ratio tables can use unit rate to solve problems. However, ratio tables can also be used to solve problems without the use of a unit rate. *Example* : In trail mix, the ratio of cups of peanuts to cups of chocolate candies is 3 to 2. how many cups of chocolate candies would be needed for 9 cups of peanuts. Show in a table. *Solution strategies:* a. students recognize that 3 cups of peanuts times 3 will give 9 cups. The amount of chocolate will also increase at the same rate (3 times) to give 6 cups of chocolate. b. Students could also find the number of cups of chocolate candies for 1 cup of peanuts by dividing both sides of the table by 3, giving 2/3 cup of chocolate for each cup of peanuts. To find the amount of chocolate needed for 9 cups of peanuts, students multiply the unit rate by 9 (9 x 2/3), giving 6 cups of chocolate.
6.RP.3c	This is students' first introduction to percents. Percentages are a rate per 100. Models, such as percent bars or 10 x 10 grids should be used to model percents. Students use percentages to find the part when given the percent, by recognizing that the whole is being divided into 100 parts and then taking a part of them (a percent). For example, to find 40% of 30, students could use a 10 x 10 grid to represent the whole (or 30). If the 30 is divided into 100 parts, the rate for one block is 0.3. Forty percent would be 40 of the blocks, or 40 x 0.3, which equals 12. Students also find the whole, given a part and the percent. For example, if 25% of the students in Mrs. Rutherford's class like chocolate ice cream, then how many students are in Mrs. Rutherford's class if 6 like chocolate ice cream? Students can reason that if 25% is 6 and 100% is 4 times the 25%, then 6 times 4 would give 24 students in Mrs. Rutherford's class. MP.1, MP.2, MP.4, MP.5, MP.7 should be emphasized.

6.NS.4	Students will find the greatest common factor of two whole numbers less than or equal to 100. For example, the greatest common factor of 40 and 16 can be found by: 1.) listing the factors of 40 (1, 2, 4, 5, 8, 10, 20, 40) and 16 (1, 2, 4, 8, 16), then taking the greatest common factor (8). Eight is also the largest number such that the other factors are relatively prime (two numbers with no common factors other than one). 2.) listing the prime factors of 40 (2 x 2 x 2 x 5) and 16 (2 x 2 x 2 x 2) and then multiplying the common factors (2 x 2 x 2 = 8) Students also understand that the greatest common factor of two prime numbers will be 1. Students use the greatest common factor and the distributive property to find the sum of two whole numbers. For example, 36 + 8 can be expressed as 4(9 + 2) = 4(11) Students find the least common multiple of two whole numbers less than or equal to twelve. For example, the least common multiple of 6 and 8 can be found by: 1.) listing the multiples of 6 (6, 12, 18, 24, 30...) and 8 (8, 16, 24, 32, 40...), then taking the least common from the list (24); or 2.) using prime factorization. a. Find the prime factors of 6 and 8 6 = 2 x 3 8 = 2 x 2 x 2 b. Find the common factors between 6 and 8. In this example, the common factor is 2 c. Multiply the common factors and any extra factors: 2 x 2 x 2 x 3 or 24 (one of the twos is in the common; the other twos and the three are extra factors.) **MP.7** should be emphasized.

UNIT VOCABULARY

coordinate plane	least common multiple	rate
equivalent ratio	ordered pair	ratio
graph	origin	ratio table
greatest common factor	prime factorization	scaling
unit price	x-axis	y-axis
unit rate	x-coordinate	y-coordinate

BIG IDEAS

ENDURING UNDERSTANDINGS	ESSENTIALS QUESTIONS Choose a few questions based on the needs of your students

Least common multiple and greatest common factor are helpful when solving real-world problems.
- A ratio is a number that relates two quantities or measures within a given situation in a multiplicative relationship (in contrast to a difference or additive relationship).The relationships and rules that govern whole numbers, govern all rational numbers.
- Making explicit the type of relationships that exist between two values will minimize confusion between multiplicative and additive situations.
- Ratios can express comparisons of a part to whole, (a/b with b ≠ 0), for example, the ratio of the number of boys in a class to the number of students in the class.
- The ratio of the length to the width of a rectangle is a part-to-part relationship.
- Understand that fractions are also part-whole ratios, meaning fractions are also ratios. Percentages are ratios and are sometimes used to express ratios.
- Both part-to-whole and part-to-part ratios compare two measures of the same type of thing. A ratio can also be a rate.
- A rate is a comparison of the measures of two different things or quantities; the measuring unit is different for each value. For example if 4 similar vans carry 36 passengers, then the comparison of 4 vans to 36 passengers is a ratio.
- All rates of speed are ratios that compare distance to time, such as driving at 45 miles per hour or jogging at 7 minutes per mile.
- Ratios use division to represent relations between two quantities.

How can you use mathematics to describe change and model real-world situations?
- How do you use equivalent rates in the real-world?
- Why would it be useful to know the greatest common factor of a set of numbers?
- Why would it be useful to know the least common multiple of a set of numbers?
- How can the distributive property help me with computation?
- What kinds of problems can I solve by using ratios?
- How can I tell if a relationship is multiplicative?
- What is the difference between a multiplicative and an additive relationship?
- What are equivalent ratios?
- What are rates?
- How are unit rates helpful in solving real-world problems?
- How are ratios and rates similar and different?
- What are percentages?
- What information do I get when I compare two numbers using a ratio?

CONNECTIONS

In **Critical Focus Area #1**, students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a wide variety of problems involving ratios and rates.

In Grade 6, students develop the foundational understanding of ratio and proportion that will be extended in Grade 7 to include scale drawings, slop and real-world percent problems.

Standards for Mathematical Practice (SMP)

MP.1 Make sense of problems and persevere in solving them
MP.2 Reason abstractly and quantitatively
MP.3 Construct viable arguments and critique the reasoning of others
MP.4 Model with mathematics
MP.5 Use appropriate tools strategically
MP.6 Attend to precision
MP.7 Look for and make use of structure (Deductive reasoning)
MP.8 Look for and express regularity in repeated reasoning (Inductive Reasoning)

UNDERSTAND RATIO CONCEPTS AND RATIO REASONING TO SOLVE PROBLEMS

CONTENT	SKILLS

6.RP.1	Understand the concept of ratio and use ratio languages to describe a ratio realtionship between two quanities	Understand the concept of ratio and use ratio languages to describe a ratio realtionship between two quanities: 1. Make part-to-whole comparisons to understand ratio 2. Make part -to-part comparisons to understand ratio 3. Express and write ratios in three forms: 6/1, 6 to 1, 6:1 4. Reduce or simiplify ratios relating the simplified values to the original numbers 5. Use models to demonstrate relationships between quantities 6. Use ratio language to describe a ratio relationship between two quantities 7. Define a ratio as a rate
6.RP.2	Understand the concept of unit rate	Understand the concept of unit rate: 1. Identify a unit rate 2. Use rate language in the context of a ratio relationship when identifying unit rate 3. Understand unit rate from various contextual situation; such as cost per item or distance per time 4. Use unit rates to solve missing value problems
6.RP.3	Use ratio and rate reasoning to solve real-world and mathematical problems	Use ratio and rate reasoning to solve real-world and mathematical problems: 1. Make tables of equivalent ratios relating quantities with whole-number measurements 2. Find missing values in tables 3. Plot the pairs of values on the coordinate plane 4. Use tables to compare ratios 5. Find a percent of a quantity as a rate per 100 6. Solve problems involving finding the whole, given a part and the percent (e.g., If 6 is 30% of a value, what is that value?)

COMPUTE FLUENTLY WITH MULTI-DIGIT NUMBERS AND FIND COMMON FACT0RS AND MULTIPLES

	CONTENT	SKILLS
6.NS.4	Find the greatest common factor of two whole numbers less than or equal to 100	Find the greatest common factor of two whole numbers less than or equal to 100 1. List factors and identify the greates common factor 2. List the prime factors and then mulitply the common factors
6.NS.4	Find the least common multiple of two whole numbers less than or equal to 12	Find the least common multiple of two whole numbers less than or equal to 12 1. List mulitples of both numbers and take the least in common from the list 2. Use prime factorization (find the prime factors of each number, find the common factors between the numbers, multiply the common factors and any extra factors)
6.NS.4	Use the distributive property to express a sum of two whole numbers with a common factor as a multiple of a sum of two whole numbers with no common factor	Use the distributive property to express a sum of two whole numbers with a common factor as a multiple of a sum of two whole numbers with no common factor 1. Find the greatest common factor 2. Using the distributive property and the GCF, rewrite the expression (e.g., 36 + 8 = 4(9 + 2) = 4(11)

UNIT RESOURCES

Common Core Model Curriculum
McGraw-Hill, **Glencoe Math**, Chapters 1-2
Georgia Math frameworks, Grade 6, Unit 2
Manipulatives
Smart Board Resources
Hands-On Standards